NUCLEON PROPERTIES IN THE PERTURBATIVE CHIRAL QUARK MODEL

V. E. LYUBOVITSKIJ[†,*], TH. GUTSCHE[†] AND AMAND FAESSLER[†]

[†]*Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany,*

[*]*Department of Physics, Tomsk State University, 634050 Tomsk, Russia,*

E-mail: Valeri.Lyubovitskij@uni-tuebingen.de, Thomas.Gutsche@uni-tuebingen.de, Amand.Faessler@uni-tuebingen.de

We apply the perturbative chiral quark model (PCQM) to analyse low-energy nucleon properties: electromagnetic form factors, meson-nucleon sigma-terms and pion-nucleon scattering. Baryons are described as bound states of valence quarks surrounded by a cloud of Goldstone bosons (π, K, η) as required by chiral symmetry. The model is based on the following guide lines: chiral symmetry constraints, fulfilment of low-energy theorems and proper treatment of sea-quarks, that is meson cloud contributions. Analytic expressions for nucleon observables are obtained in terms of fundamental parameters of low-energy pion-nucleon physics (weak pion decay constant, axial nucleon coupling constant, strong pion-nucleon form factor) and of only one model parameter (radius of the nucleonic three-quark core). Our results are in good agreement with experimental data and results of other theoretical approaches.

1 Introduction

Hadron models set up to understand the structure of the nucleon should respect the constraints imposed by chiral symmetry. Spontaneous and explicit chiral symmetry breaking require the existence of the pion whose mass vanishes in the limit of zero current quark mass. In turn, nucleon observables must receive contributions from pion loops.

We recently suggested [1] a baryon model, the perturbative chiral quark model (PCQM), which includes relativistic quark wave functions and confinement and respects chiral symmetry. The PCQM was successfully applied to σ-term physics [1] and extended to the study of electromagnetic properties of the nucleon [2] and πN scattering [3]. Although the Lagrangian for this model fulfills the $SU(2) \times SU(2)$ current algebra, it is in general nontrivial to identify the explicit dynamics which leads to well-known predictions for various observables. For instance, S-wave πN scattering at threshold is such a process which should be correctly described by a nucleon model involving the pion cloud.

2 Perturbative Chiral Quark Model

2.1 Effective Lagrangian and zeroth order properties

Following considerations are based on the perturbative chiral quark model (PCQM), a relativistic quark model suggested in [4] and extended in [1] for the study of low-energy properties of baryons. Similar models have also been studied in Refs. [5]. The PCQM is based on the effective, chirally invariant Lagrangian $\mathcal{L}_{inv}$ [1]

$$\begin{aligned}\mathcal{L}_{inv} = \bar\psi\Big\{ i \not\partial - \gamma^0 V(r) - S(r) \left[\frac{U + U^\dagger}{2} + \gamma^5 \frac{U - U^\dagger}{2}\right]\Big\}\psi \\ + \frac{F^2}{4}\,\mathrm{Tr}[\,\partial_\mu U\,\partial^\mu U^\dagger\,] \end{aligned} \tag{1}$$

where ψ is the quark field, U is the chiral field and $F = 88$ MeV is the pion decay constant in the chiral limit [1,6]. The quarks move in a self-consistent field, represented by scalar $S(r)$ and vector $V(r)$ components of an effective static potential providing confinement. The interaction of quarks with Goldstone bosons is introduced on the basis of the nonlinear σ-model [7]. We define the chiral field as $U = \exp[i\hat\Phi/F]$ using the *exponentional parametrization* of the nonlinear σ-model, where $\hat\Phi$ is the matrix of pseudoscalar mesons. When treating mesons fields as small fluctuations we do perturbation theory in the expansion parameter $1/F$. Usually, the Lagrangian (1) is linearized with respect to the field $\hat\Phi$. Such an approximation is valid if we consider processes without free pions (e.g. nucleon mass shift due to the pion cloud) or with a single external pion field (e.g. pion-nucleon form factor). The resulting approximate chiral invariance of the linearized Lagrangian [1,8] guarantees both a conserved axial current (or PCAC in the presence of the meson mass term) and the Goldberger-Treiman relation. However, when we study πN scattering the quadratic term in the expansion of the chiral field U should be kept. With the additional nonlinear term we reproduce [3] the model-independent result for the πN S-wave scattering lengths as derived by derived by Weinberg and Tomozawa [9].

Here we concentrate on the application of the PCQM to the meson-nucleon σ-terms [1] and the electromagnetic properties of the nucleon [2]. We use the effective Lagrangian $\mathcal{L}_{eff} = \mathcal{L}_{inv}^{lin} + \mathcal{L}_{\chi SB}$ including the linearized chiral-invariant term $\mathcal{L}_{inv}^{lin}$ and a mass term $\mathcal{L}_{\chi SB}$ which explicitly breaks chiral symmetry

$$\mathcal{L}_{inv}^{lin} = \bar\psi[i \not\partial - S(r) - \gamma^0 V(r)]\psi + \frac{1}{2}(\partial_\mu\hat\Phi)^2 - \bar\psi S(r) i\gamma^5\frac{\hat\Phi}{F}\psi,$$

$$\mathcal{L}_{\chi SB} = -\bar{\psi}\mathcal{M}\psi - \frac{B}{8}\mathrm{Tr}\{\hat{\Phi},\,\{\hat{\Phi},\,\mathcal{M}\}\}, \tag{2}$$

where $\mathcal{M} = \mathrm{diag}\{\hat{m}, \hat{m}, m_s\}$ is the mass matrix of current quarks (here and in the following we restrict to the isospin symmetry limit with $m_u = m_d = \hat{m}$); B is the low-energy constant which measures the vacuum expectation value of the scalar quark densities in the chiral limit [10]. We rely on the standard picture of chiral symmetry breaking [10] and for the masses of pseudoscalar mesons we use the leading term in their chiral expansion (i.e. linear in the current quark mass): $M_\pi^2 = 2\hat{m}B$, $M_K^2 = (\hat{m}+m_s)B$, $M_\eta^2 = \frac{2}{3}(\hat{m}+2m_s)B$. Meson masses satisfy the Gell-Mann-Oakes-Renner and the Gell-Mann-Okubo relation $3M_\eta^2 + M_\pi^2 = 4M_K^2$. In the evaluation we use the following set of QCD parameters: $\hat{m}$= 7 MeV, $m_s/\hat{m}$=25 and $B = M_{\pi^+}^2/(2\hat{m})$=1.4 GeV. Introduction of the electromagnetic field A_μ into the PCQM is accomplished by standard minimal substitution into Eq. (2).

To describe the properties of baryons which are modelled as bound states of valence quarks surrounded by a meson cloud we formulate perturbation theory. In our approach the mass (energy) m_N^{core} of the three-quark core of the nucleon is related to the single quark energy $\mathcal{E}_0$ by $m_N^{core} = 3 \cdot \mathcal{E}_0$. For the unperturbed three-quark state we introduce the notation $|\phi_0>$ with the appropriate normalization $<\phi_0|\phi_0>= 1$. The single quark ground state energy $\mathcal{E}_0$ and wave function (w.f.) $u_0(\vec{x})$ are obtained from the Dirac equation

$$[-i\vec{\alpha}\vec{\nabla} + \beta S(r) + V(r) - \mathcal{E}_0]u_0(\vec{x}) = 0. \tag{3}$$

The quark w.f. $u_0(\vec{x})$ belongs to the basis of potential eigenstates (including excited quark and antiquark solutions) used for expansion of the quark field operator $\psi(x)$. Here we restrict the expansion to the ground state contribution with $\psi(x) = b_0 u_0(\vec{x})\exp(-i\mathcal{E}_0 t)$, where b_0 is the corresponding single quark annihilation operator. In Eq. (3) the current quark mass is not included to simplify our calculational technique. Instead we consider the quark mass term as a small perturbation.

For a given form of the potentials $S(r)$ and $V(r)$ the Dirac equation (3) can be solved numerically. Here, for the sake of simplicity, we use a variational *Gaussian ansatz* [1] for the quark wave function given by the analytical form:

$$u_0(\vec{x}) \;=\; N\,\exp\left[-\frac{\vec{x}^2}{2R^2}\right]\,\begin{pmatrix} 1 \\ i\rho\,\dfrac{\vec{\sigma}\vec{x}}{R} \end{pmatrix}, \tag{4}$$

where $N = [\pi^{3/2}R^3(1+3\rho^2/2)]^{-1/2}$ is a constant fixed by the normalization condition $\int d^3x\, u_0^\dagger(x)\, u_0(x) \equiv 1$; χ_s, χ_f, χ_c are the spin, flavor and color quark wave functions, respectively. Our Gaussian ansatz contains two model

parameters: the dimensional parameter R and the dimensionless parameter ρ. The parameter ρ can be related to the axial coupling constant g_A calculated in zeroth-order (or 3q-core) approximation:

$$g_A = \frac{5}{3}\left(1 - \frac{2\rho^2}{1+\frac{3}{2}\rho^2}\right) = \frac{5}{3}\frac{1+2\gamma}{3}, \tag{5}$$

where $\gamma = 9g_A/10 - 1/2$. The parameter R can be physically understood as the mean radius of the three-quark core and is related to the charge radius $< r_E^2 >^P_{LO}$ of the proton in the leading-order (LO) approximation as [1]

$$< r_E^2 >^P_{LO} = \frac{3R^2}{2}\frac{1\,+\,\frac{5}{2}\rho^2}{1\,+\,\frac{3}{2}\rho^2} \,=\, R^2\left(2 - \frac{\gamma}{2}\right). \tag{6}$$

In our calculations we use the value g_A=1.25 as obtained in ChPT [6]. We therefore have only one free parameter, R. In the numerical evaluation R is varied in the region from 0.55 fm to 0.65 fm.

2.2 Perturbation theory and nucleon mass

Following the Gell-Mann and Low theorem we define the mass shift of the nucleonic three-quark ground state Δm_N due to the interaction with Goldstone mesons as

$$\Delta m_N \,\doteq\, {}^N\!\!<\phi_0|\sum_{n=1}^{\infty}\frac{i^n}{n!}\int i\delta(t_1)\,d^4x_1\ldots d^4x_n\,T[\mathcal{L}_I(x_1)\ldots\mathcal{L}_I(x_n)]|\phi_0>^N_c \tag{7}$$

where $\mathcal{L}_I = -S(r)\bar{\psi}\,i\gamma^5(\hat{\Phi}/F)\,\psi$ is the interaction Lagrangian between mesons and quarks treated as a perturbation and subscript "c" refers to contributions from connected graphs only. We evaluate Eq. (7) at one loop with $o(1/F^2)$ using Wick's theorem and the appropriate propagators. For the quark field we use a Feynman propagator for a fermion in a binding potential. By restricting the summation over intermediate quark states to the ground state we get

$$\begin{aligned} iG_\psi(x,y) &= <\phi_0|T\{\psi(x)\bar{\psi}(y)\}|\phi_0> \\ &\to u_0(\vec{x})\bar{u}_0(\vec{y})\exp[-i\mathcal{E}_0(x_0-y_0)]\theta(x_0-y_0). \end{aligned} \tag{8}$$

For meson fields we use the free Feynman propagator for a boson field. Superscript "N" in Eq. (7) indicates that the matrix elements are projected on the respective nucleon states.

The total nucleon mass including one-loop corrections is given by

$$m_N = 3(\mathcal{E}_0 + \gamma\hat{m}) + \sum_\Phi d_N^\Phi\Pi(M_\Phi^2);\; d_N^\pi = \frac{171}{400}, d_N^K = \frac{6}{19}d_N^\pi, d_N^\eta = \frac{1}{57}d_N^\pi, \tag{9}$$

where d_N^Φ are the recoupling coefficients defining the partial contribution of the π, K and η-meson cloud to the mass shift of the nucleon. The contribution of a finite current quark mass to the nucleon mass shift is taken into account perturbatively (for details see [1]). The self-energy operator $\Pi(M_\Phi^2)$ is given by

$$\Pi(M_\Phi^2) \, = \, - \biggl(\frac{g_A}{\pi F}\biggr)^2 \int\limits_0^\infty \frac{dp\, p^4}{w_\Phi^2(p^2)} \, F_{\pi NN}^2(p^2) \tag{10}$$

where $w_\Phi(p^2) = \sqrt{M_\Phi^2 + p^2}$ is the meson energy with $p = |\vec{p}|$ and $F_{\pi NN}(p^2)$ is the πNN form factor normalized to unity at zero recoil ($p^2 = 0$):

$$F_{\pi NN}(p^2) = \exp\biggl(-\frac{p^2 R^2}{4}\biggr)\biggl\{1 \, + \, \frac{p^2 R^2}{8}\biggl(1 \, - \, \frac{5}{3g_A}\biggr)\biggr\}. \tag{11}$$

2.3 Meson nucleon sigma-terms and Feynman-Hellmann theorem

The scalar density operators S_i^{PCQM} $(i = u, d, s)$, relevant for the calculation of the meson-baryon sigma-terms in the PCQM, are defined as the partial derivatives of the model χSB Hamiltonian $\mathcal{H}_{\chi SB} = -\mathcal{L}_{\chi SB}$ with respect to the current quark mass of i-th flavor m_i:

$$S_i^{PCQM} \doteq \frac{\partial \mathcal{H}_{\chi SB}}{\partial m_i} \, = \, S_i^{val} \, + \, S_i^{sea}. \tag{12}$$

S_i^{val} is the set of valence-quark operators coinciding with the ones obtained from the QCD Hamiltonian. The set of sea-quark operators S_i^{sea} arises from the pseudoscalar meson mass term [1]. To calculate meson-baryon sigma-terms we perform the perturbative expansion for the matrix element of the scalar density operator S_i^{PCQM} between unperturbed 3q-core states and then project it onto the respective baryon wave functions.

As an example, the expression for $\sigma_{\pi N}$ is given by

$$\sigma_{\pi N} = \hat{m} \, < p|S_u^{PCQM} \, + S_d^{PCQM}|p >= 3\gamma\hat{m} \, + \, \sum_\Phi d_N^\Phi \Gamma(M_\Phi^2) \tag{13}$$

where the first term of the right-hand side of Eq. (13) corresponds to the valence quark, the second to the sea quark contribution. The vertex function $\Gamma(M_\Phi^2)$ is related to the partial derivative of the self-energy operator $\Pi(M_\Phi^2)$ with respect to $\hat{m}$:

$$\Gamma(M_\Phi^2) \, = \, \hat{m} \, \frac{\partial}{\partial \hat{m}} \, \Pi(M_\Phi^2). \tag{14}$$

Using Eqs. (9), (13) and (14) we directly prove the Feynman-Hellmann theorem $\sigma_{\pi N} = \hat{m} \cdot \partial m_N / \partial \hat{m}$.

3 Results

3.1 Meson-nucleon sigma-terms

We start our numerical analysis with the πN sigma-term. First, we restrict to the $SU(2)$ flavor picture. Numerically, the contribution of the valence quarks to the πN sigma-term is 13.1 MeV, the contribution of sea quarks at order $o(M_\pi^2)$ is 66.9 ± 5.7 MeV. Here and in the following the error bars are due to variation of the range parameter R of the quark wave function (4) from 0.55 to 0.65 fm. Taking into account higher-order contributions of the sea quarks we have the following result for the πN sigma-term $\sigma_{\pi N}^{\pi}$ (superscript π refers to the SU(2) flavor picture) of

$$\sigma_{\pi N}^{\pi} = 43.3 \pm 4.4 \,\mathrm{MeV}. \tag{15}$$

The contributions of kaon and η-meson loops, $\sigma_{\pi N}^{K}$ and $\sigma_{\pi N}^{\eta}$ (superscripts K and η refer to the respective meson cloud contribution) are significantly suppressed relative to the pion cloud and to the valence quark contributions due to the energy denominators in the structure integrals. The same conclusion regarding the suppression of K and η-meson loops was obtained in the cloudy bag model [11]. Numerically, kaon and η-meson cloud contributions are $\sigma_{\pi N}^{K} = 1.7 \pm 0.4$ MeV and $\sigma_{\pi N}^{\eta} = 0.023 \pm 0.006$ MeV. For the πN sigma-term we have the following final value:

$$\sigma_{\pi N} = \sum_{\Phi} \sigma_{\pi N}^{\Phi} = 45 \pm 5 \,\mathrm{MeV}. \tag{16}$$

Our result for the πN sigma-term is in perfect agreement with the value of $\sigma_{\pi N} \simeq 45$ MeV deduced by Gasser, Leutwyler and Sainio [12] using dispersion-relation techniques and exploiting the chiral symmetry constraints.

Next we discuss our prediction for the strangeness content of the nucleon y_N which is defined in the PCQM as

$$y_N = \frac{2 < p|S_s^{PCQM}|p >}{< p|S_u^{PCQM} + S_d^{PCQM}|p >}. \tag{17}$$

The direct calculation of the strange-quark scalar density $< p|S_s^{PCQM}|p >$ is completely consistent with the indirect one applying the Feynman-Hellmann theorem $< p|S_s^{PCQM}|p >= \partial m_N / \partial m_s$. The small value of $y_N = 0.076 \pm 0.012$ in our model is due to the suppressed contributions of kaon and η-meson loops. Our prediction for y_N is smaller than the value $y_N \simeq 0.2$ obtained in [12] from an analysis of experimental data on πN phase shifts. On the other hand, our prediction is quite close to the result obtained in the cloudy bag model $y_N \approx 0.05$ [11].

For the KN sigma-terms we obtain

$$\sigma^u_{KN} = 340 \pm 37\,\mathrm{MeV} \ \text{ and } \ \sigma^d_{KN} = 284 \pm 37\,\mathrm{MeV}, \tag{18}$$

which within uncertainties is consistent with values deduced in HBChPT and lattice QCD (see discussion in Ref. [1]). Hopefully, future DAΦNE experiments at Frascati will allow for a determination of the KN sigma-terms and hence for a better knowledge of the strangeness content of the nucleon.

3.2 Electromagnetic properties of the nucleon

We extended our model analysis to the description of basic electromagnetic properties of the nucleon. Formal details can be found in Ref. [2]. We start with results for the magnetic moments of nucleons, μ_p and μ_n. For our set of parameters we obtain:

$$\mu_p = 2.62 \pm 0.02, \quad \mu_n = -2.02 \pm 0.02, \quad \text{and} \quad \frac{\mu_n}{\mu_p} = -0.76 \pm 0.01. \tag{19}$$

The leading order (LO, three-quark core) μ_p^{LO}, μ_n^{LO} and next-to-leading order (NLO, meson cloud and finite current quark mass) contributions μ_p^{NLO}, μ_n^{NLO} to the magnetic moments are given by

$$\begin{aligned} &\mu_p^{LO} = 1.8 \pm 0.15, \quad \mu_n^{LO} \equiv -\frac{2}{3}\mu_p^{LO}, \qquad\qquad (20)\\ &\mu_p^{NLO} = \mu_p - \mu_p^{LO} = 0.82 \pm 0.13, \quad \mu_n^{NLO} = \mu_n - \mu_n^{LO} = -0.82 \pm 0.08. \end{aligned}$$

For the electromagnetic nucleon radii we obtain

$$\begin{aligned} r_E^p &= 0.84 \pm 0.05\,\mathrm{fm}, \qquad < r^2 >_E^n = -0.036 \pm 0.003\,\mathrm{fm}^2, \qquad (21)\\ r_M^p &= 0.82 \pm 0.02\,\mathrm{fm}, \qquad r_M^n = 0.85 \pm 0.01\,\mathrm{fm}. \end{aligned}$$

The LO contributions to the charge radius of the proton (see Eq. (6)) and to the magnetic radii of proton and neutron are dominant

$$r_E^{p;\,LO} \; = \; 0.77 \pm 0.06\,\mathrm{fm}, \quad r_M^{p;\,LO} \equiv r_M^{n;\,LO} \; = \; 0.73 \pm 0.06\,\mathrm{fm}. \tag{22}$$

For the neutron charge radius squared we get the observed (negative) sign, but its magnitude is smaller than the experimental value. As in the naive $SU(6)$ quark model, the LO contribution to the neutron charge radius is zero and only one-loop diagrams give nontrivial contributions to this quantity:

$$< r^2 >_E^n = -0.036 \pm 0.003 \ \mathrm{fm}^2. \tag{23}$$

In Table 1 we summarize our results for the static electromagnetic properties of the nucleon in comparison to experimental data. Results on the Q^2-dependence of the electromagnetic form factors can be found in Ref. [2].

Table 1. Static nucleon properties.

Quantity	Our Approach	Experiment
μ_p	2.62 $\pm$ 0.02	2.793
μ_n	-2.02 $\pm$ 0.02	-1.913
μ_n/μ_p	-0.76 $\pm$ 0.01	-0.68
r_E^p (fm)	0.84 $\pm$ 0.05	0.86 $\pm$ 0.01
$< r^2 >_E^n$ (fm^2)	-0.036 $\pm$ 0.003	-0.119 $\pm$ 0.004
r_M^p (fm)	0.82 $\pm$ 0.02	0.86 $\pm$ 0.06
r_M^n (fm)	0.85 $\pm$ 0.01	0.88 $\pm$ 0.07

Acknowledgments

This work was supported by the Deutsche Forschungsgemeinschaft (DFG, grant FA67/25-1). V.E.L. thanks the organizers for the invitation.

References

1. V.E. Lyubovitskij, T. Gutsche, A. Faessler and E.G. Drukarev, *Phys. Rev.* D **63**, 54026 (2001).
2. V.E. Lyubovitskij, T. Gutsche and A. Faessler, hep-ph/0105043.
3. V.E. Lyubovitskij, T. Gutsche, A. Faessler, A. Rusetsky and R. Vinh Mau (in preparation).
4. T. Gutsche and D. Robson, *Phys. Lett.* B **229**, 333 (1989). T. Gutsche, Ph. D. Theses, Florida State University, 1987 (unpublished).
5. A.W. Thomas, *Adv. Nucl. Phys.* **13**, 1 (1984); E. Oset, R. Tegen and W. Weise, *Nucl. Phys. A* **426**, 456 (1984); S.A. Chin, *Nucl. Phys. A* **382**, 355 (1982).
6. J. Gasser, M.E. Sainio and A.B. Švarc, *Nucl. Phys.* B **307**, 779 (1988).
7. M. Gell-Mann and M. Lévy, *Nuovo Cim.* **16**, 1729 (1960).
8. A. W. Thomas, *J. Phys. G* **7**, L283 (1981).
9. S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966); Y. Tomozawa, *Nuovo Cim. A* **46**, 707 (1966).
10. S. Weinberg, *Physica A* **96**, 327 (1979); J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984); *Nucl. Phys.* B **250**, 465 (1985).
11. R. E. Stuckey and M. C. Birse, Nucl. Phys. **B250**, 465 (1985). *J. Phys. G* **23**, 29 (1997).
12. J. Gasser, H. Leutwyler and M. E. Sainio, *Phys. Lett.* B **253**, 252,260 (1991).